Evaluate $\int \cos^{-1} x \, dx$. **SCORE: _____ / 4 PTS**

u	dv
$\cos^{-1} x$	1
$-\frac{1}{\sqrt{1-x^2}}$	x
-1	$\frac{x}{\sqrt{1-x^2}}$
0	$-\sqrt{1-x^2}$

$$x\cos^{-1}x + \int \frac{x}{\sqrt{1-x^2}}\,dx \leftarrow \begin{array}{l} u = 1-x^2 \\ du = -2x\,dx \end{array}$$

(1) (1)

$$\int -\frac{1}{2}\frac{1}{\sqrt{u}}\,du = -\sqrt{u} = -\sqrt{1-x^2}$$

(1)

OR

$$= x\cos^{-1}x - \sqrt{1-x^2} + C$$

(1)

(−1) IF YOU FORGOT "+C"

Evaluate $\int \frac{(\ln x)^2}{x^2}\, dx$.

SCORE: _____ / 4 PTS

u		dv
$(\ln x)^2$	$+$	x^{-2}
$\frac{2\ln x}{x}$		$-x^{-1}$
$2\ln x$		$-x^{-2}$
$\frac{2}{x}$	$-$	x^{-1}
2	$+$	x^{-2}
0		$-x^{-1}$

$-x^{-1}(\ln x)^2 - 2x^{-1}\ln x - 2x^{-1} + C$

(1) (1½) (1½)

(−1) IF YOU FORGOT "+C"

Evaluate $\int e^{-2x}\cos 4x\,dx$. **SCORE: _____ / 5 PTS**

u	dv
$\cos 4x$	$+\ e^{-2x}$
$-4\sin 4x$	$-\ -\frac{1}{2}e^{-2x}$
$-16\cos 4x$	$+\ \frac{1}{4}e^{-2x}$

$$\int e^{-2x}\cos 4x\,dx = -\frac{1}{2}e^{-2x}\cos 4x + e^{-2x}\sin 4x - \int 4e^{-2x}\cos 4x\,dx$$

(1) (1) (1)

$$5\int e^{-2x}\cos 4x\,dx = -\frac{1}{2}e^{-2x}\cos 4x + e^{-2x}\sin 4x$$ (1)

$$\int e^{-2x}\cos 4x\,dx = -\frac{1}{10}e^{-2x}\cos 4x + \frac{1}{5}e^{-2x}\sin 4x + C$$

(1)

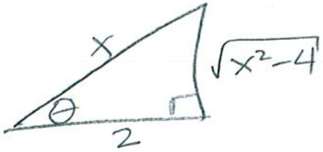
(−1) IF YOU FORGOT "+C"

Evaluate $\int (x^2-4)^{\frac{3}{2}}\,dx$.

SCORE: _____ / 7 PTS

$\int 4^{\frac{3}{2}}\left(\frac{x^2}{4}-1\right)^{\frac{3}{2}}dx$

$\frac{x^2}{4}=\sec^2\theta$



(1) $x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$= \int 8(\sec^2\theta-1)^{\frac{3}{2}} \cdot 2\sec\theta\tan\theta\,d\theta$

$= 16\int \tan^3\theta \cdot \sec\theta\tan\theta\,d\theta$

(1/2) POINT EXCEPT AS NOTED

(−1) IF YOU FORGOT "+C"

$= 16\int \sec\theta\tan^4\theta\,d\theta$

$= 16\int \sec\theta(\sec^2\theta-1)^2\,d\theta$

$= 16\int (\sec^5\theta - 2\sec^3\theta + \sec\theta)\,d\theta$

$= 16\left[\frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}\int\sec^3\theta\,d\theta - 2\int\sec^3\theta\,d\theta + \int\sec\theta\,d\theta\right]$

$$= 4\sec^3\theta\tan\theta - 20\int\sec^3\theta\, d\theta + 16\int\sec\theta\, d\theta$$

$$= 4\sec^3\theta\tan\theta - 20\left(\tfrac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|)\right)$$

$$+ 16\ln|\sec\theta + \tan\theta| + C$$

$$= 4\sec^3\theta\tan\theta - 10\sec\theta\tan\theta + 6\ln|\sec\theta + \tan\theta| + C$$

$$= 4\left(\frac{x}{2}\right)^3\frac{\sqrt{x^2-4}}{2} - 10\left(\frac{x}{2}\right)\frac{\sqrt{x^2-4}}{2} + 6\ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + C$$

$$= \frac{1}{4}x^3\sqrt{x^2-4} - \frac{5}{2}x\sqrt{x^2-4} + 6\ln\left|x + \sqrt{x^2-4}\right| + C$$

Evaluate $\int \sin^2 x \cos^5 x \, dx$ ☆ IF YOU USED THE REDUCTION FORMULA, YOU MAY EARN FULL CREDIT IF YOU WRITE AN ALGEBRAIC PROOF THAT YOUR FINAL ANSWER IS EQUIVALENT

SCORE: _____ / 4 PTS

$u = \sin x \rightarrow du = \cos x \, dx$ (1)

(1) $\int u^2 (1-u^2)^2 \, du = \int (u^2 - 2u^4 + u^6) \, du$ ($\frac{1}{2}$)

($\frac{1}{2}$) $= \frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7 + C$

(1) $= \frac{1}{3}\sin^3 x - \frac{2}{5}\sin^5 x + \frac{1}{7}\sin^7 x + C$

(−1) IF YOU FORGOT "+C"

Prove the reduction formula $\int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$.

SCORE: _____ / 6 PTS

<u>NOTE: You must show how to get this formula.</u>

<u>You will receive 0 credit if your "proof" is differentiating both sides of the equation.</u>

u	dv
$\sec^{n-2} u$	$\sec^2 u$
$(n-2)\sec^{n-2} u \tan u$	$\tan u$

$\int \sec^n u \, du = \sec^{n-2} u \tan u - (n-2) \int \sec^{n-2} u \tan^2 u \, du$ (2)

$= \sec^{n-2} u \tan u - (n-2) \int \sec^{n-2} u (\sec^2 u - 1) \, du$ (3)

$= \sec^{n-2} u \tan u - (n-2) \int \sec^n u \, du + (n-2) \int \sec^{n-2} u \, du$

$(n-1) \int \sec^n u \, du = \sec^{n-2} u \tan u + (n-2) \int \sec^{n-2} u \, du$ (1)

$\int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$